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# On the dynamic susceptibility of the bulging domain wall model of polycrystalline magnetic oxides

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The static initial magnetic susceptibility due to domain wall motion in a polycrystalline magnetic oxide has been explained by Globus *et al.* [3] using a model of a bulging domain wall inside grains of uniform diameter, $D$. The present work deals with the dynamic response of this model by solving the equation of motion of such a wall. The resultant solution reproduces Globus' relation for the static case and further shows that the dispersion frequency is $\sim D^{-1}$ for small grainsizes and $\sim D^{-2}$ for large grain sizes.

## 1. Introduction

Rado, Wright, and Emerson [2] were the first to point out the contribution of domain wall motion to the susceptibility of magnetic oxides. Since then Brown and Gravel [3] and in particular Globus [4] have done extensive work on the domain wall motion. Using a simple model of a polycrystalline material consisting of spherical grains of uniform diameter, $D$, inside which there is only one 180° domain wall pinned to the grain boundary, Globus has calculated the grain-size dependence of the static initial magnetic susceptibility [1] and the relaxation frequency [4]. In this work we examine the dynamic response of this model by solving the equation of motion of such a domain wall (see Fig. 1).

The equation of motion† can be written as

$$m\frac{\partial^2 z}{\partial t^2} + \beta\frac{\partial z}{\partial t} = \gamma\nabla^2 z + 2M_s H_x e^{i\omega t} \qquad (1)$$

where

$z(r, t)$ = displacement of the plane domain wall perpendicular to its plane,
$\gamma$ = domain wall energy per unit area,
$\beta$ = a damping constant, which includes damping due to eddy current and spin rotation,
$m$ = effective mass of the domain wall,
$M_s$ = the saturation magnetization,
$H_x$ = component of the magnetic field along the magnetization in the bulging half of the grain,
and $\omega$ = the angular frequency of the applied magnetic field

The boundary condition to be satisfied is

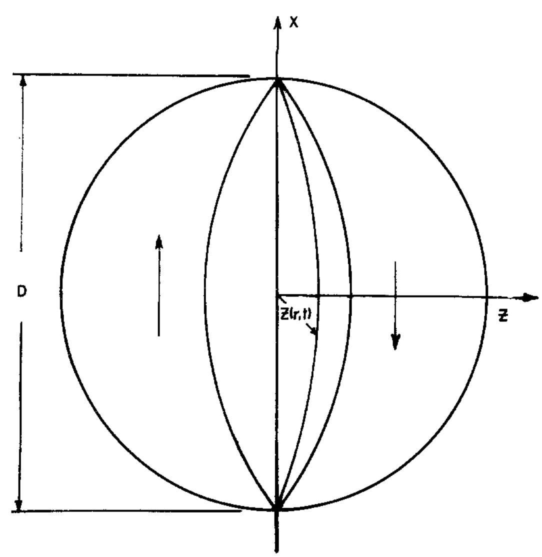


*Figure 1* Globus' model of a 180° domain wall inside a grain, which bulges under a magnetic field.

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†Equation 1 does not account for the free pole formation due to the bulging domain wall.

$$z(D/2, t) = 0 \tag{2}$$

and a solution to Equation 1 can be written as

$$z = T_k(t) J_k(\mu_k r/D) \tag{3}$$

where $J_k$s are Bessel functions.

Combining Equations 1 and 3;

$$\frac{4\mu_k^2\gamma}{D^2} T_k(t) + \beta \dot{T}_k(t) + m\ddot{T}_k(t) = \frac{4M_s H_x}{\mu_k J_1(\mu_k)} e^{i\omega t} \tag{4}$$

Solution to Equation 4 has been given by Rossel [7] and $z(r, t)$ can be obtained as

$$z(r,t) = \sum_{k=1}^{\infty} \frac{M_s H_x D^3 \exp\left[i(\omega t - \delta_k)\right] J_0\left(\dfrac{2\mu_k r}{D}\right)}{\gamma \mu_k^3 J_1(\mu_k)\left[\left(1 - \dfrac{\omega^2}{\omega_{rk}^2}\right)^2 + \left(\dfrac{\omega}{\omega_{ck}}\right)^2\right]^{1/2}} \tag{5}$$

where

$$\omega_{rk} = \left(\frac{4\mu_k^2\gamma}{mD^2}\right)^{1/2}$$

$= $ resonant frequency when $\beta = 0$

$$\omega_{ck} = \frac{4\mu_k^2\gamma}{\beta D^2}$$

$=$ relaxation frequency when $m = 0$,

and $\tan\delta_k = (\omega/\omega_{ck})/(1 - \omega^2/\omega_{rk}^2)$.

The volume swept by the bulging of a wall is given by

$$V = \int_0^{z(r=0)} \pi r^2 \, dz = \frac{\pi}{2} D^2 \sum_{k=1}^{\infty} \frac{J_1(\mu_k)}{\mu_k} T_k(t) \tag{6}$$

and the magnetic susceptibility is

$$\chi = \frac{12 M_s V}{\pi D^3 H} \tag{7}$$

Since the direction of magnetization inside a domain is randomly oriented with respect to the applied magnetic field inside a polycrystalline material

$$|H_x| = \tfrac{1}{3}|H| \tag{8}$$

Combining Equations 5, 6, and 8 the real ($\chi'$) and imaginary ($\chi''$) parts of the susceptibility can be written as

$$\chi' = \sum_{k=1}^{\infty} \frac{2M_s^2 D\left(1 - \dfrac{\omega^2}{\omega_{rk}^2}\right)}{\mu_k^4 \gamma \left[\left(1 - \dfrac{\omega^2}{\omega_{rk}^2}\right)^2 + \left(\dfrac{\omega}{\omega_{ck}}\right)^2\right]} \tag{9}$$

$$\chi'' = \sum_{k=1}^{\infty} \frac{2M_s^2 D(\omega/\omega_{ck})}{\mu_k^4 \gamma \left[\left(1 - \dfrac{\omega^2}{\omega_{rk}^2}\right)^2 + \left(\dfrac{\omega}{\omega_{ck}}\right)^2\right]} \tag{10}$$

## 2. Static initial magnetic susceptibility

Under static or low frequency conditions, $\omega \to 0$, and Equations 9 and 10 reduce to

$$\chi' = \mu' - 1 = \frac{M_s^2 D}{\gamma} \sum_{k=1}^{\infty} \frac{2}{\mu_k^4} = a_1 \frac{M_s^2 D}{\gamma} \tag{11}$$

$$\chi'' = \mu'' = \frac{M_s^2 \beta\omega D^3}{\gamma^2} \sum_{k=1}^{\infty} \frac{1}{2\mu_k^6} = a_2 \frac{M_s^2 \beta\omega D^3}{\gamma^2} \tag{12}$$

The sums in Equations 11 and 12 are rapidly converging; $a_1 = 0.062$ and $a_2 = 0.00254$. Equation 11 is essentially the same as the one derived by Globus *et al.* [1] except for the numerical factor, $a_1$, which in their case was three times the value obtained in the present calculation. This arises from the averaging procedure we have used (see Equation 8).

It may be of interest to note that a grain-size distribution or presence of multidomain grains (as happens in practice) does not affect Equation 11 except for the factor, $a_1$, since $\chi' \sim D$. But $\chi''$ or magnetic losses due to the domain wall motion at low motion at low frequencies is a relatively strong function of grain size distribution since $\chi'' \sim D^3$ and is heavily weighed by the largest grain size rather than by the average grain size.

## 3. Dynamic susceptibility and resonance

As the frequency of applied magnetic field is increased, various modes of vibration of the circular doman wall play increasingly important roles. As the resonance (or relaxation) frequency of each mode is approached the contribution of that mode to the real and imaginary parts of the magnetic susceptibility increases. The real and imaginary parts of the susceptibility can be written as sums of various modes of vibrations (see Equations 9 and 10).

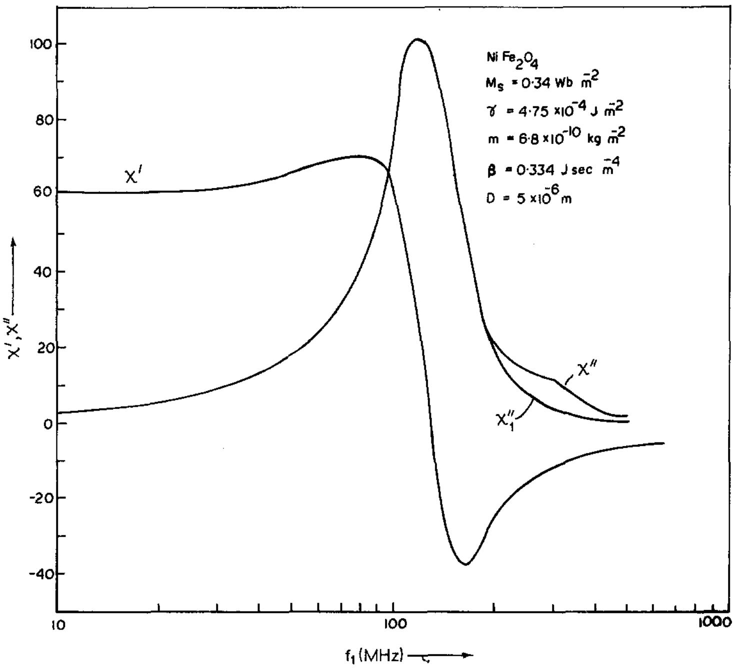


*Figure 2* Variation of real ($\chi'$) and imaginary ($\chi''$) parts of the magnetic susceptibility with frequency.

$$\chi' = \sum_{k=1}^{\infty} \chi'_k \tag{13}$$

$$\chi'' = \sum_{k=1}^{\infty} \chi''_k \tag{14}$$

Fig. 2 shows the frequency spectrum of the real and imaginary parts of the magnetic susceptibility of a nickel ferrite with 5 $\mu$m grain size. It can be seen the $\chi'_1$ and $\chi''_1$ substantially determine the value of the sums in Equations 13 and 14. The higher modes of vibration slightly modify the high frequency tail of the susceptibility spectra. Hence we assume it to be sufficient to treat the behaviour of $\chi'_1$ and $\chi''_1$ to describe the dynamic magnetic susceptibility.

The peak in the imaginery part of the magnetic susceptibility gives the frequency of maximum magnetic loss. Let that frequency be $f_r (\omega_r = 2\pi f_r)$. Then setting

$$\left[\frac{\partial \chi''_1}{\partial \omega}\right]_{\omega=\omega_r} = 0 \tag{15}$$

leads to the equation

$$\frac{6m^2\omega_r^2}{\beta^2} = \eta - 1 + \sqrt{(1 - 2\eta + 4\eta^2)} \tag{16}$$

where

$$\eta = \xi/D^2$$

and

$$\xi = \frac{8\mu_1^2\gamma_m}{\beta^2},$$

an intrinsic parameter determined by the property of the material. For small mass and large grain sizes, when $\eta \to 0, f_r$ is given by

$$f_r = \frac{2\mu_1^2\gamma}{\pi\beta D^2} = \frac{\omega_{c1}}{2\pi} \tag{17}$$

and for small grain sizes, when $\eta \to \infty, f_r$ is given by

$$f_r = \frac{1}{\pi D}\sqrt{\left(\frac{\mu_1^2\gamma}{m}\right)} = \frac{\omega_{r1}}{2\pi} \tag{18}$$

It can be seen that for large grain sizes $f_r \sim D^{-2}$ and for small grain sizes $f_r \sim D^{-1}$. Globus [7] observed that $f_r \sim D^{-2}$ (these observations are consistent with our results as they were taken on materials with a relatively large grain size i.e. $> 8\,\mu$m) whereas Turk [8] observed that $f_r \sim D^{-1}$ (grain sizes used by Turk were less than 10 $\mu$m).

The effective mass, $m$, and the damping constant, $\beta$, for a magnetic oxide are given by [6].

$$m = \frac{\mu_0\gamma}{2\nu^2 A} \tag{19}$$

$$\beta = \frac{2\pi\mu_0\gamma\lambda}{\nu^2 A} \tag{20}$$

where $\nu$ = gyromagnetic constant, $\lambda$ = relaxation frequency in the Landau-Liftshitz equation, and $A$ = exchange constant.

Substituting Equations 19 and 20 in Equation 16.

$$f_r = \lambda[\tfrac{2}{3}\{\eta - 1 + \sqrt{(1 - 2\eta + 4\eta^2)}\}]^{1/2}$$
$$= \lambda F(\eta) \tag{21}$$

Fig. 3 gives the variation of $F(\eta)$ with grain size for various values of $\xi$.

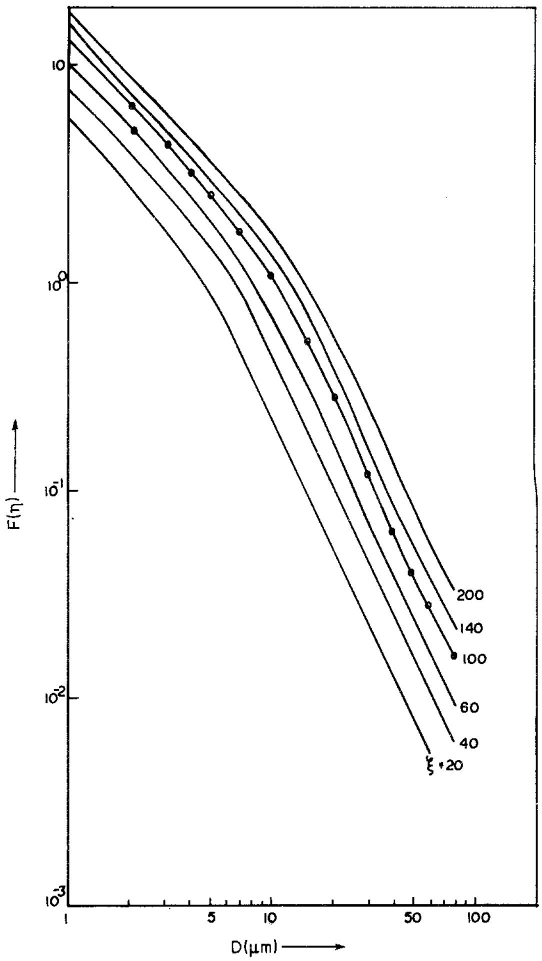


*Figure 3* Variation of $F(\eta)$ with grain size.

## 4. Resonances or relaxation

It has been argued by Globus [4] that in nickel ferrite there is no resonance but only relaxation due to domain wall motion if the grain size is uniform. His conclusion was based on his observation that there was no peak in the real part of susceptibility spectra. However, he observed a peak in the real part of susceptibility spectra if the grain-size distribution was wide. It is pertinent to calculate the frequency, $f_m$, at which a peak in $\chi'$ spectra will occur. This is given by,

$$f_m = \frac{\omega_{r1}}{2\pi}\sqrt{\left(1 - \frac{\omega_{r1}}{\omega_{c1}}\right)} \tag{22}$$

It can be seen that if $\omega_{r1}/\omega_{c1} \geqslant 1$, there will be no peak in the $\chi'$ spectra. However, minima in the $\chi'$ spectra will occur at a frequency given by

$$f'_m = \frac{\omega_{r1}}{2\pi}\sqrt{\left(1 + \frac{\omega_{r1}}{\omega_{c1}}\right)} \tag{23}$$

It can be shown that below a certain grain size, $D_c$, $\omega_{r1}/\omega_{c1} < 1$ and a resonance peak is observed in the magnetic susceptibility spectra. Above this grain size no maximum in the real part of magnetic susceptibility is expected although a minimum is observed. This critical grain size is given by

$$D_{c'} = \sqrt{\left(\frac{4\mu_1^2\gamma m}{\beta^2}\right)} \tag{24}$$

For nickel ferrite this critical grain size can be calculated to be $8\,\mu$m. Since Globus material contained grains with sizes larger than $8\,\mu$m, no peak in the $\chi'$ spectra is expected and this is in agreement with his data. When the grain size distribution is wide, we explain the peak in the $\chi'$ spectra by involving the presence of grains of size less than the critical grain size.

## 5. Summary

The results of the present calculation can be summarized as follows:

(1) The static initial magnetic susceptibility of a polycrystalline magnetic oxide $\sim M_s^2 D/\gamma$.

(2) The dispersion frequency of the domain wall motion $\sim D^{-2}$ for large grain sizes and $\sim D^{-1}$ for small grain sizes.

(3) There is a critical grain size below which a resonance behaviour and above which a relaxation behaviour is expected.


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